Finite Element Convergence Results

Theorem 7.5 (Deterministic or L^∞ RHS)

Let Assumptions 1 and 2 hold, and let $V^h \subset H_0^1(D)$ be the space of piecewise linear FEs with respect to a shape-regular triangulation $\mathscr{T}_h$ (see Appendix B). Furthermore, suppose that $f \in L^\infty(\Omega; L^2(D))$ (in particular includes deterministic f), $a_{\min}^{-1/2} a_{\max}^{1/2} \in L^q(\Omega;\mathbb{R})$ and $C_2 \in L^r(\Omega;\mathbb{R})$ with $q, r \in [1,\infty]$ s.t. $\frac{1}{p} = \frac{1}{q} + \frac{1}{r} \le 1$, then

$$\|u - u_h\|_{L^p(\Omega; H_0^1(D))} \le ch \|f\|_{L^\infty(\Omega; L^2(D))}.$$

$$|u - u_h(\cdot,\omega)|_{H^1(D)} \le C\, C_2(\omega) \sqrt{\frac{a_{\max}(\omega)}{a_{\min}(\omega)}}\, \|f(\cdot,\omega)\|_{L^2(D)}\, h$$

$$\mathbb{E}\left[|u - u_h(\cdot,\omega)|^p_{H^1(D)} \right]^{1/p} \le C \operatorname{esssup} \|f(\cdot,\omega)\|_{L^2(D)}\, h\, \mathbb{E}\left[C_2^p \left(\frac{a_{\max}}{a_{\min}}\right)^{p/2}\right]^{1/p}$$

Hölder

$$\le C \|f\|_{L^\infty(\Omega, L^2(D))}\, h\, \underbrace{\left\| \sqrt{\frac{a_{\max}}{a_{\min}}} \right\|_{L^q(\Omega;\mathbb{R})}}_{<\infty} \underbrace{\|C_2\|_{L^r(\Omega;\mathbb{R})}}_{<\infty}$$