Seminar “Multiscale Methods”

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Natural or engineered materials often contain two or more key constituents, arranged in a heterogeneous structure varying at different scales. Such materials are desirable because their macroscopic properties can be superior to the properties of the individual constituents. It is even possible to explicitly design them for a particular purpose by changing the composition of the constituents. An example are carbon fibre composites for lightweight structures and vehicles. The mathematical modelling of such heterogeneous or composite materials typically leads to PDEs with highly oscillating coefficients. Direct numerical solution of such problems with traditional methods, such as finite elements is computationally expensive. Just to compute the correct qualitative behaviour, the mesh resolution would need to be sufficiently high to capture all the fine scale variation.

In this seminar, we will study several multiscale numerical methods that do not suffer from this drawback, including the multiscale finite element method, the generalized multiscale finite element method, the heterogeneous multiscale method, the variational multiscale method, and the localization orthogonal decomposition method. Each student will present a key publication on multiscale numerical methods with an aim to cover each of the individual types with at least one presentation.

Pre-requisites. Basic knowledge of PDEs, Sobolev spaces and FE methods is required.

Administrative Matters

Due to current situation around the COVID-19 pandemic the seminar will (at least initially) be held in an online form on the heiCONF platform. We will also create a moodle page for interaction and to share material. Possibly, we will be able to have the final presentations in a more traditional format, but that is very uncertain at this point.

Initial Meeting (online) Tue, April 21, 2020, 11.15 (**please use the following link**):

`https://heiconf.uni-heidelberg.de/sch-66r-awy`

Further dates and logistics regarding the selection of topics and how to interact with us will be communicated then. If you have any questions prior to this meeting please email

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Key Publications (non exclusive)

[1] A. Abdulle, E. Weinan, B. Engquist, and E. Vanden-Eijnden, The heterogeneous multiscale method, *Acta Numerica* **21**:1–87, 2012.

[2] G. Allaire and R. Brizzi, A multiscale finite element method for numerical homogenization, *Multiscale Model. Simul. (SIAM)* **4**:790–812, 2005.

[3] Y. Efendiev, J. Galvis and T.Y. Hou, Generalized multiscale finite element methods (GMsFEM), *J. Comput. Phys.* **251**:116–135, 2013.

[4] Y. Efendiev, J. Galvis, G. Li, and M. Presho, Generalized multiscale finite element methods: Oversampling strategies, *Int. J. Multiscale Comput. Eng.* **12**(6), 2014.

[5] Y. Efendiev and T.Y. Hou, *Multiscale Finite Element Methods: Theory and Applications*, Springer, New York, 2009.

[6] T.J.R. Hughes, G.R. Feijo, L. Mazzei and J.-B. Quincy, The variational multiscale method – a paradigm for computational mechanics, *Comput. Meth. Appl. Mech. Engrg.* **166**:3-24, 1998.

[7] T.J.R. Hughes and G. Sangalli, Variational multiscale analysis: the fine-scale Greens function, projection, optimization, localization, stabilized methods, *SIAM J. Numer. Anal.* **45**:539–557, 2007.

[8] R. Kornhuber, D. Peterseim, and H. Yserentant, An analysis of a class of variational multiscale methods based on subspace decomposition, *Math. Comput.* **87**(314):2765–2774, 2018.

[9] A. Malqvist and D. Peterseim, Localization of elliptic multiscale problems, *Math. Comput.* **83**(290):2583–2603, 2014.

[10] P. Ming and P. Zhang, Analysis of the heterogeneous multiscale method for elliptic homogenization problems, *J. American Math. Soc.* **18**(1):121–156, 2005.