High-dimensional Approximation and Applications in UQ
(A Specialist Numerical Analysis Lecture)

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Many physical models from the natural sciences and from engineering involve sources of uncertainty that affect their outputs. An example are variations in the orientation of layers of carbon fibres occurring naturally in the manufacturing process of aircraft wings, which lead to difficulties in making precise predictions about the locations and types of possible defects that may occur. The goal of uncertainty quantification is to use mathematical and computational methods to account for such uncertainties, and to understand how they propagate through to model outputs. The research of our group focuses on developing innovative numerical methods to efficiently quantify uncertainty. We apply these techniques to tackle data-driven, large-scale problems that are typically modelled in the form of differential equations. From a mathematical, or more specifically numerical, point of view this leads to high-dimensional approximation and integration problems. As such, tackling high-dimensional problems has recently become a very active research field in numerical analysis.

As motivation, the course begins with a brief introduction to Uncertainty Quantification and to why such problems have high dimensionality. We will then give an overview of some of the key technologies that have recently been developed to efficiently tackle high-dimensional problems, and the necessary mathematical tools to rigorously analyse these methods.

In particular, we will focus on

- Uncertainty Quantification and the “Curse of Dimensionality”;
- Monte Carlo methods (in particular, multilevel Monte Carlo);
- Quasi-Monte Carlo methods;
- Sparse grids — quadrature and approximation;
- Adaptive methods and best N-term approximation; and
- Low-rank tensor approximation.

Pre-requisites. Basic knowledge of PDEs, Sobolev spaces and FE methods is required.

Administrative Matters

Due to current situation around the COVID-19 pandemic the lecture will (at least initially) be held in an online form on the heiCONF platform. We will also create a moodle page for interaction and to share material. Possibly, we will be able to have the later lectures in a more traditional format, but that is very uncertain at this point.

Initial Meeting (online) Mon, April 20, 2020, 9.15 (**please use the following link**):

`https://heiconf.uni-heidelberg.de/sch-zcd-jn3`

Further dates and logistics regarding the selection of topics and how to interact with us will be communicated then. If you have any questions prior to this meeting please email

`r.scheichl@uni-heidelberg.de` or `a.gilbert@uni-heidelberg.de`

References

Since this course draws on several different areas of recent and very active research, there is no single text that covers all of the content. As such, we will take content from the following references.

[1] G. J. Lord, C. E. Powell and T. Shardlow. *An Introduction to Computational Stochastic PDEs*. Cambridge University Press, Cambridge, UK, 2014.

[2] M. B. Giles. Multilevel Monte Carlo methods. *Acta Numerica*, **24**, 259–328, 2015.
`https://doi.org/10.1017/S096249291500001X`

[3] J. Dick, F. Y. Kuo, and I. H. Sloan. High-dimensional integration: The quasi-Monte Carlo way. *Acta Numerica*, **22**, 133–288, 2013.
`https://doi.org/10.1017/S0962492913000044`

[4] H-J. Bungartz and M. Griebel. Sparse grids. *Acta Numerica*, **13**, 147–169.
`https://doi.org/10.1017/S0962492904000182`

[5] R. G. Ghanem and P. Spanos. *Stochastic Finite Elements*. Springer, New York, 1991.

[6] W. Hackbusch. Numerical tensor calculus. *Acta Numerica*, **23**, 651–742, 2014.
`https://doi.org/10.1017/S0962492914000087`